Role of liquid compressional viscosity in the dynamics of a sonoluminescing bubble

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The well-known Rayleigh-Plesset (RP) equation is the basis of almost all hydrodynamical descriptions of single-bubble sonoluminescence (SBSL). A major deficiency of the RP equation is that it accounts for viscosity of an incompressible liquid and compressibility, separately. By removing this approximation, a new modification of the RP equation is presented considering effect of compressional viscosity of the liquid. This modification leads to addition of a new viscous term to the traditional bubble boundary equation. Influence of this new term in the dynamics of a sonoluminescing bubble has numerically been studied considering effects of heat transfer at the bubble wall, nonequilibrium evaporation and condensation of water vapor, chemical reactions, and diffusion of the reactions products in the liquid. The results show that the new term has a significant damping role in the bubble motion at the end of collapse and during the rebounds, so that its consideration dramatically reduces amplitude of the afterbounces. Dependence of this new damping mechanism on the driving pressure amplitude and on the ambient radius has been investigated. The results indicate that the more intense the collapse, the more important the damping of the liquid compressional viscosity.

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I. INTRODUCTION

Non-linear radial oscillations of a small gas bubble in a liquid under the influence of a high amplitude ultrasound field concentrate energy into the bubble to produce picosecond light pulses. This phenomenon is known as single-bubble sonoluminescence (SBSL) [1,2], which was discovered by Gaitan and Crum in 1990 [3]. After this discovery, a large number of experimental as well as theoretical papers were published describing different characteristics of such unusual phenomenon, including duration of pulse width (40–350 ps) [4–6], intensity and spectrum of emitted light [7–10], its dependence to ambient parameters [11–13] and dissolved gas in the liquid [14], experimental phase diagrams, [15–18], and different criteria that a sonoluminescing bubble must simultaneously satisfy for stability [19–22].

In nearly all existing theoretical descriptions of the sonoluminescence characteristics, radial dynamics of the bubble is described by the well-known Rayleigh-Plesset (RP) equation. Several different forms of the RP equation are available in the literature derived by many authors [23–30]. One of the most popular ones is the equation derived by Keller and Miksis [31]:

$$\left(1-\frac{\dot{R}}{C}\right)R\ddot{R}+\frac{3}{2}\left(1-\frac{\dot{R}}{3C}\right)\dot{R}^2=\frac{R}{\rho C}\frac{d}{dt}(P_l-P_a)+\left(1+\frac{\dot{R}}{C}\right)\frac{P_l-P_a-P_0}{\rho}, \tag{1}$$

where R, C, P_0, P_a, and ρ are the bubble radius, liquid sound speed, ambient pressure, driving pressure, and liquid density, respectively. Equation (1) must be supplemented by a boundary condition equation at the bubble interface to relate the liquid pressure, P_l, to the gas pressure inside the bubble. In all existing theoretical analysis of the nonlinear bubble dynamics, the following incompressible boundary equation has been used for this purpose:

$$P_l=P_g-4\mu\frac{\dot{R}}{R}-\frac{2\sigma}{R}, \tag{2}$$

where P_g, μ, and σ are the gas pressure at the bubble interface, liquid shear viscosity, and surface tension, respectively. It should be mentioned that the difference of Eq. (1) with the other first order forms of the RP equation arises from the terms proportional to $\dot{R}/C$. Prosperetti and Lezzi showed that there is a one-parameter family of equations describing the bubble motion in the first order approximation of the compressibility and Eq. (1) belongs to this family [32].

There is a common part between all forms of the RP equation. That is the incompressible boundary condition Eq. (2). We note that this equation has been used even in the second-order equations derived by previous authors [33,34]. Equation (2) is derived under a specific approximation. That is the incompressibility assumption of the liquid motion at the bubble interface. We emphasize that all effects of the liquid compressibility in the RP equation arise from the liquid motion around the bubble, but not from the bubble boundary condition equation. In fact, in all forms of the RP equation, a compressible equation [Eq. (1)] has been supplemented by an incompressible boundary condition equation [Eq. (2)]. This means that all forms of the RP equation account for viscosity of an incompressible liquid and compressibility, separately.

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For strongly driven bubbles, such as sonoluminescing bubbles, the incompressibility approximation in Eq. (2) is reasonably applicable for all times before the collapse, due to the incompressibility characteristics of the bubble motion. However, at the collapse time, when the bubble becomes highly compressed, the incompressibility assumption is completely violated. Therefore, it is expected that the neglected compressibility effect at the bubble boundary becomes quite important. Since the sonoluminescence radiation is produced at the end of collapse, modification of Eq. (2) with the compressibility effect is essential for a better description of sonoluminescence characteristics.

In this paper, we present a new modification of the RP equation considering effect of compressional viscosity of the liquid. This modification leads to a new viscous term including two coefficients of viscosity, which is added to the traditional bubble boundary equation [Eq. (2)]. The influence of this term on the dynamics of a sonoluminescing bubble has numerically been investigated using an ODE hydrochemical model. The results clearly emphasize the importance of the liquid compressional viscosity at the collapse time and during the sonoluminescence radiation.

II. COMPRESSIBLE BUBBLE BOUNDARY EQUATION

To derive the compressible bubble boundary equation, we assume that the motions of the bubble interface and the surrounding liquid are always spherically symmetric. The continuity equation and the radial component of the stress tensor, τ_{rr}, can be written as [35]

$$\frac{1}{\rho}\left[\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right]=-\frac{\partial u}{\partial r}-\frac{2u}{r}=-\nabla\cdot\vec{\mathbf{u}}, \tag{3}$$

$$\tau_{rr}=-p+\lambda\nabla\cdot\vec{\mathbf{u}}+2\mu\left(\frac{\partial u}{\partial r}\right), \tag{4}$$

where ρ, p, and $\vec{\mathbf{u}}=u\hat{\mathbf{r}}$ are density, pressure, and velocity vector, respectively. Also, λ is second coefficient of viscosity. Inserting $\partial u/\partial r$ from Eq. (3) into Eq. (4) yields

$$\tau_{rr}=-p+(\lambda+2\mu)\nabla\cdot\vec{\mathbf{u}}-4\frac{\mu u}{r}. \tag{5}$$

From Eq. (3), the velocity divergence can be written as

$$\nabla\cdot\vec{\mathbf{u}}=-\frac{1}{\rho}\frac{d\rho}{dt}=-\frac{1}{\rho C^2}\frac{dp}{dt}. \tag{6}$$

where the sound speed, C, is defined as $C^2=dp/d\rho$. The boundary continuity requirement at the bubble interface is

$$\tau_{rr}(\text{liquid})_R=\tau_{rr}(\text{gas})_R+2\frac{\sigma}{R}. \tag{7}$$

Applying Eq. (5) for the gas and the liquid parts of Eq. (7) and neglecting the gas viscosity similar to Eq. (2), due to its smallness relative to the liquid viscosity, leads to

$$P_l+4\frac{\mu\dot{R}}{R}-(\lambda+2\mu)\vec{\nabla}\cdot\vec{\mathbf{u}}\Big|_R=P_g-2\frac{\sigma}{R}. \tag{8}$$

Substituting the divergence of liquid velocity at the bubble wall from Eq. (6) into Eq. (8) yields

$$P_l+\frac{\lambda+2\mu}{\rho C^2}\frac{dP_l}{dt}=P_g-4\frac{\mu\dot{R}}{R}-2\frac{\sigma}{R}. \tag{9}$$

Equation (9) is the modified form of Eq. (2) including effect of the liquid compressibility. We note that Eqs. (1) and (9) provide a new modification of the RP equation, which accounts for viscosity of a compressible liquid. The added new term of Eq. (9) includes simultaneous effects of the liquid compressibility and viscosity. Since, Eq. (9) is more complete than Eq. (2), it is more appropriate for description of the bubble dynamics, especially during the sonoluminescence radiation.

III. BUBBLE INTERIOR EVOLUTION

To quantify effects of the new viscous term on the bubble dynamics, evolution of the gas pressure at the bubble interface, P_g, must be specified. The model that we use here for this purpose is the recent hydrochemical ODE model of Lohse *et al.* [36–38], which accounts for effects of heat and mass transfer at the bubble interface as well as chemical reactions. This model appropriately describes various experimental phase diagrams and provides a good agreement with the complete direct numerical simulation of Storey and Szeri [39].

We describe an argon bubble in water, which is the final state of an air SL bubble, according to the rectified diffusion hypothesis [40]. The bubble contents are the noncondensable argon gas, water vapor, and the main chemical reactions products at the end of collapse, which are H, H_2, OH, O_2, and O. The number of particles inside the bubble changes with time, due to diffusion at the bubble wall and the chemical reactions. The gas pressure is obtained by Eqs. (3)–(16) of Ref. [38], which are not repeated here. These equations along with the bubble dynamics equations are the set of equations that totally describe the evolution of the bubble characteristics. Under these circumstances, in this work, time variations of the bubble properties have numerically been calculated for both the new and the old RP equations.

IV. NUMERICAL ANALYSIS

The calculations were carried out for a periodic driving pressure: $P_a(t)=P_a\sin(\omega t)$, with $\omega=2\pi\times 26.5$ kHz. The constants and parameters were set for water at room temperature, $T_0=293.15$ K, and atmospheric ambient pressure, $P_0=1.0$ atm; i.e., $\rho=998.0$ kg/m^3, $C=1483.0$ m/s, $\mu=1.01\times 10^{-3}$ kg/ms, and $\sigma=0.0707$ kgs^{-2} [41]. The second coefficient of viscosity of water at room temperature was set $\lambda=3.43\times 10^{-3}$ kg/ms [42].

Figures 1–4 illustrate the results for $P_a=1.35$ atm and $R_0=4.5$ μm. Similar values for these phase parameters have been reported in several recent experimental works [15–18].

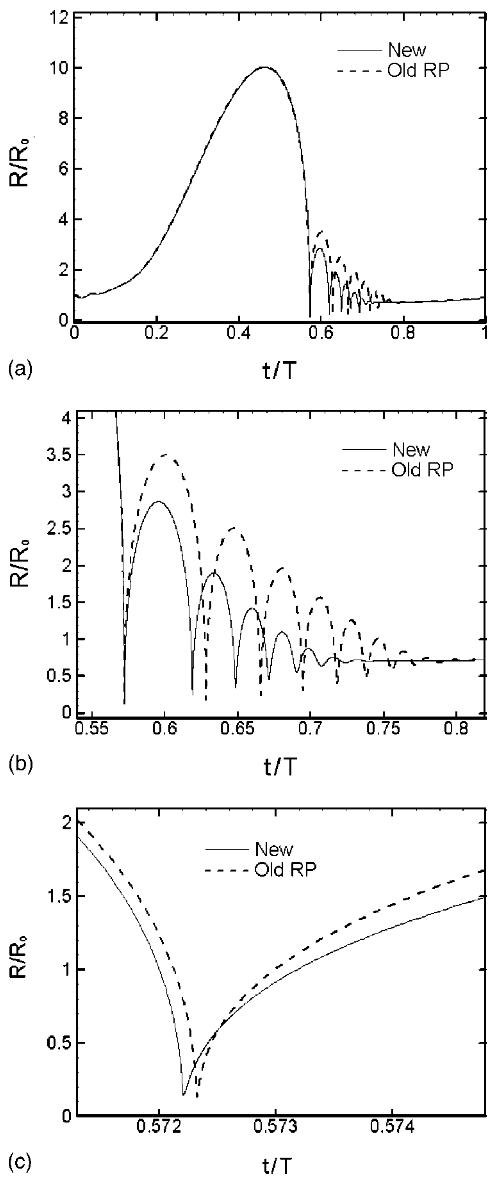

FIG. 1. Time variations of the bubble radius for a sonoluminescing bubble with parameters space $R_0=4.5\ \mu\mathrm{m}$ and $P_a=1.35$ atm, according to the new (solid) and the old RP (dashed) equations. Graph (a) shows the bubble evolution during a complete period (T). Graphs (b) and (c) show the radius variations during the rebounds and at the end of collapse, respectively.

Figure 1 shows the calculated radius-time curves for the new and the old RP equations. It is seen that the new viscous term considerably affects the bubble evolution at the end of collapse and during the rebounds. Since the bubble motion is quite compressible at the end of collapse, the new term, which resulted from the liquid compressibility, is important in this time interval. It exhibits a damping role and its consideration reduces strength of the collapse. This damping appears in the increase of the minimum radius for the new equation relative to that of the old one [about 10% in Fig. 1(c)]. Also, the reduction of the collapse intensity remarkably diminishes the amplitude of the afterbounces, which is accompanied with the decrease in time interval between two successive rebounds [Fig. 1(b)].

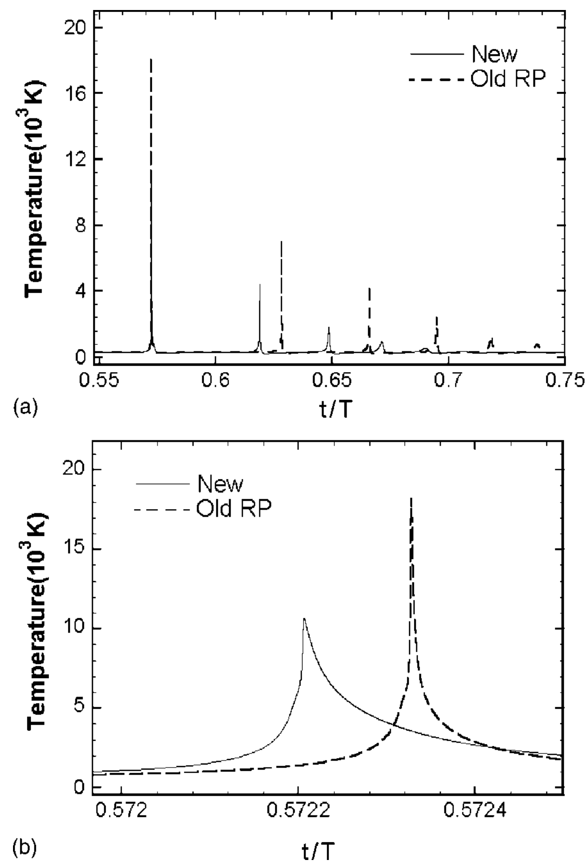

FIG. 2. The evolution of the bubble temperature during the rebounds (a), and at the end of the collapse (b), according to the new (solid) and the old RP (dashed) equations for the same parameters and constants as in Fig. 1.

In many experimental data for the bubble radius available in the literature (e.g., Fig. 1 of Ref. [29] and Figs. 14 and 16 of Ref. [1]), it is seen that the bubble rebounds are rapidly damped, which is in contrast with what various old RP forms predict. Moss *et al.* suggested a damping term arisen from the gas compressibility to solve this problem [43]. The influence of their suggested gas-based term is very similar to the damping effect of liquid compressional viscosity (compare Fig. 1 with Figs. 2–6 of Ref. [43]. This similarity indicates that the modified RP equation presented here should be in a better agreement with experimental data than the traditional forms.

In Fig. 2, the gas temperature evolution during the rebounds and at the end of collapse are shown. The damping feature of the new term is seen in the considerable decrease of the maximum temperature (about 40%) as well as in the reduction of the magnitudes of the secondary peaks [Fig. 2(a)]. We note that the pulse width of the main peak increases with the addition of the new term [Fig. 2(b)].

Figure 3 shows the evolution of total number of particles species inside the bubble, N_{tot}, for both the new and the old RP cases. The number of molecules considerably increases in the expansion region due to evaporation of vapor molecules from the surrounding liquid into the bubble. During the collapse, the vapor molecules inside the bubble rapidly condense to the liquid. Chemical reactions only occur at the

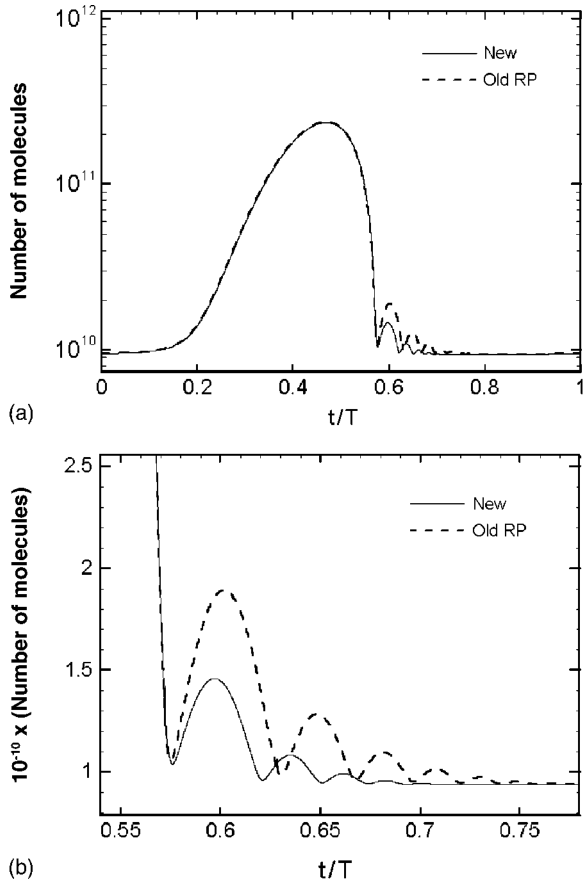

FIG. 3. Time variations of the total number of particles species inside the bubble, according to the new (solid) and the old RP (dashed) equations. Graphs (a) and (b) show the evolution in a complete period and during the rebounds, respectively. The parameters and constants are the same as in Fig. 1.

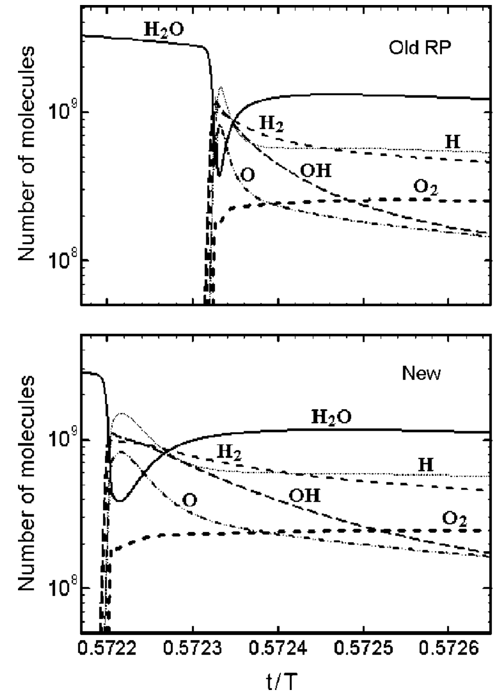

FIG. 4. The number of molecules of water vapor and the reaction products at the end of the collapse, for the new and the old RP cases for the same parameters and constants as in Fig. 1.

end of collapse, when the bubble temperature is enough high to destroy chemical bands of the vapor molecules [44]. It is seen that a considerable difference between the two cases in the number of molecules appears during the bubble rebounds. This difference gradually disappears as the bubble rebounds weaken.

Influence of the new term on the evolution of water vapor molecules and the reaction products at the end of collapse has been shown in Fig. 4. It shows that considerable differences exist between the number of molecules of the two cases. We note that the peaks in the new case are wider than those of the old RP case.

The illustrated damping of the compressional viscosity at the end of collapse and during the rebounds affects some of the previous theoretical analyses of nonlinear bubble dynamics. Two special examples are stability limits of strongly collapsing bubbles and the magnitude of light emission from the SL bubbles. The bubble instability is very sensitive to the collapse intensity and the rebounds thereafter. Also, the amount of light emission and the spectrum of the emitted light strongly depend on the maximum temperature achievable. As we showed in Figs. 1–4, these characteristics are dramatically affected by the addition of the new term.

In Figs. 5 and 6, we have presented dependence of the maximum temperature and the compression ratio $s = R_{max}/R_{min}$ on the variation of phase parameters (P_a and R_0). The different values of P_a, corresponding to a specific value of R_0, can be experimentally obtained by proper adjusting of the dissolved gas concentration in the liquid [1].

Figure 5 shows the maximum temperature and compression ratio as a function of the driving pressure amplitude. As it is expected, the collapse intensity increases when the pressure amplitude is amplified. It is seen that the addition of the new term to the RP equation always decreases both the maximum temperature and the compression ratio. The damping effect is more considerable for higher driving pressures (about 100% difference in the maximum temperature for $P_a = 1.5$ atm).

Figure 6 represents the dependence of the maximum temperature and the compression ratio on the ambient radius. The damping feature of the new term also exists here. Similar to Fig. 5, this figure also shows that the difference between the two cases increases when the collapse intensity is enhanced by the reduction of the ambient radius.

V. CONCLUSIONS

All traditional forms of the RP equation have a common deficiency. They account for viscosity of an incompressible liquid and compressibility, separately. This deficiency was removed by introducing a more complete bubble boundary equation containing effect of liquid compressional viscosity. The derived equation has a new term including two coefficients of viscosity. This term is important at the collapse time of a strongly driven bubble, e.g., a sonoluminescing bubble. The new term has a damping role and its consideration reduces the collapse intensity and the amplitude of the after-

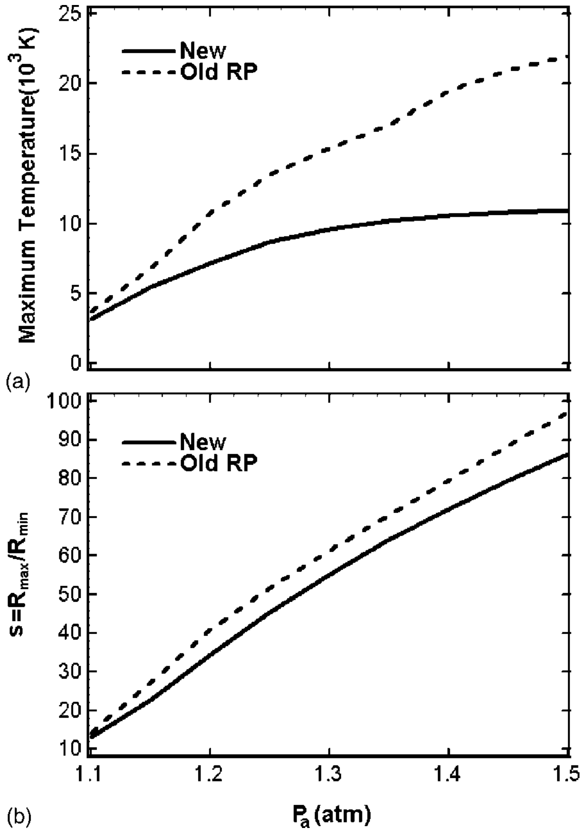

FIG. 5. The maximum temperature of the bubble (a), and the compression ratio $s=R_{max}/R_{min}$ (b), as a function of the driving pressure amplitude for the new (solid) and the old RP (dashed) equations. The equilibrium radius is fixed ($R_0=4.5$ μm). Other constants are the same as in Figs. 1–4.

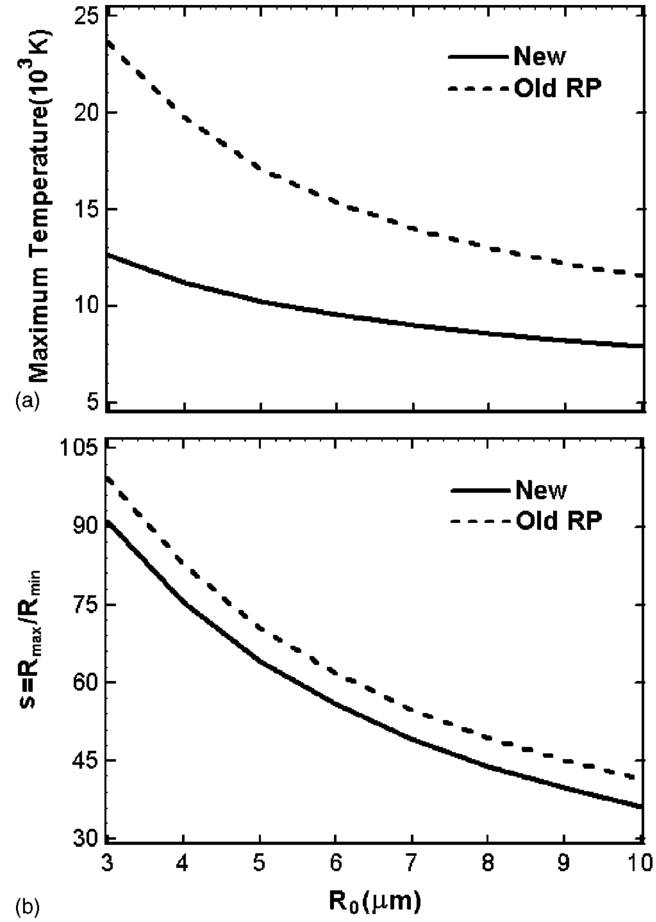

FIG. 6. The dependence of the maximum temperature (a), and the compression ratio (b), on the ambient radius for both the new (solid) and the old RP (dashed) equations. The pressure amplitude is fixed ($P_a=1.35$ atm) and the other constants are the same as in Figs. 1–4.

bounces. The more intense the collapse, the more significant the damping of the liquid compressional viscosity.

The new effect also results in a lower maximum temperature of the bubble. This should lead to a decrease in the amount of light emission as well as to a significant change in the spectrum of the emitted light. Most of previous experimental reports show that the SBSL spectrum is best approximated by black body radiation [45–47]. However, the calculated lower peak temperatures present a problem for theoretical description of the measured Planck spectrum and for prediction of its maximum, which is clearly displayed in the experiments [46,47]. This problem may be resolved by considering nonuniform energy focusing inside the bubble. As indicated in nonuniform simulations [8,48,49], the occurrence of shock waves inside a sonoluminescing bubble considerably raises the peak temperature at the bubble center, which can diminish the low temperature problem.

On the other hand, the findings of this paper should affect previous theoretical predictions of the bubble stability limits [19–22,37,38]. This may restrict the bubble phase parameters (P_a and R_0) to the values that push upward the maximum temperature. In previous works [19,22], it is shown that the shape stability restricts the ambient radius of a high pressure driven bubble ($P_a \gtrsim 1.5$ atm) to the smaller sizes ($R_0 \lesssim 4.0$ μm). The maximum temperature increases with the reduction in the ambient radius (Fig. 6), and this may resolve the temperature problem for strongly driven bubbles.

By comparing the results of this work with the similar results of Moss *et al.* [43], the derived new equation seems to be in a better agreement with experimental data than the traditional RP forms.

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